

Falsification of Mannheim's conformal gravity

May 15, 2013

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Abstract

We show that Mannheim's conformal gravity, whose potential has a term proportional to $1/r$ and another term proportional to r , doesn't reduce to Newtonian gravity at short distances. Therefore, despite the claim that it successfully explains galaxy rotation curves, it seems falsified by numerous Cavendish-type experiments performed at laboratories on Earth whose work haven't found any deviations from Newton's theory. Moreover, when Mannheim used his potential to fit the galaxy rotation curve, he used the Newtonian formula to calculate the effects of the term proportional to $1/r$, not the conformal gravity one. So, he lacked consistency. After all, he would not have been able to use the conformal gravity one either since it deviates so much from the Newtonian one, which the conformal gravity one should reduce to. We also give a couple of other similar reasons why Mannheim's conformal gravity is wrong. For example, the gravitational potential of conformal gravity doesn't reduce to the Newtonian one even in short distance limit.

1 Introduction

Recently, Mannheim's conformal gravity theory attracted much attention as an alternative to dark matter and dark energy [1, 2, 3]. However, so far, the only way its validity could be tested was through cosmological considerations. In this paper, we suggest that Mannheim's conformal gravity is already falsified by numerous Cavendish-type experiments on Earth. One of our ideas is that Mannheim's conformal gravity theory predicts that the gravitational force due to an object depends on its mass distribution even in the case that it has a spherical symmetric mass distribution. (i.e. the case in which the mass density only depends on the distance from the center of the body.) For example,

Newtonian gravity predicts that we can calculate the gravitational force due to earth, as if all the mass of earth were at the center of earth. This is not true in case of the conformal gravity; the gravitational force *heavily* depends on the mass distribution.

In section 2 and 3, we introduce and review the conformal gravity. In sections 4, 5, 6, and 7 we give four arguments why conformal gravity is falsified. Among our arguments in this paper, the one in section 5 must be regarded as most definitive.

2 Mannheim's conformal gravity

Instead of Einstein-Hilbert action, in conformal gravity, we have the following action.

$$\begin{aligned} S &= -\alpha_g \int \sqrt{-g} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \\ &= -2\alpha_g \int \sqrt{-g} [R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2] \end{aligned} \quad (1)$$

where $C_{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor, and α_g is a purely dimensionless coefficient. Adding this to the action of matter, and varying with respect to the metric, one can obtain the conformal gravity version of the Einstein equation.

3 The metric solution in conformal gravity

The following derivation closely follows [3]. (See in particular, (9), (13), (14) and (16) in their paper.) In case there is a spherical symmetry in the distribution of the mass, one can write the metric as follows:

$$ds^2 = -B(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_2 \quad (2)$$

Plugging this to the conformal gravity version of the Einstein equation, and assuming that all the matter is inside the radius r_0 , Mannheim and Kazanas obtained:

$$B(r > r_0) = 1 - \frac{2\beta}{r} + \gamma r \quad (3)$$

$$\nabla^4 B(r) = f(r) \quad (4)$$

The solution is given by:

$$2\beta = \frac{1}{6} \int_0^{r_0} dr' r'^4 f(r') \quad (5)$$

$$\gamma = -\frac{1}{2} \int_0^{r_0} dr' r'^2 f(r') \quad (6)$$

where

$$f(r) \equiv \frac{3}{4\alpha_g B(r)} (T_0^0 - T_r^r) \quad (7)$$

without any approximation whatsoever.

4 Falsification by Cavendish-type experiments

If we ignore T_r^r in the above equation, as it is small, set $T_0^0 = \rho$, and use $B(r) \approx 1$, we get:

$$\frac{2\beta}{r} = \frac{1}{r} \left(\frac{1}{8\alpha_g} \int_0^{r_0} dr' r'^4 \rho \right) \quad (8)$$

Compare this with Newtonian case, which is following:

$$\frac{2\beta}{r} = \frac{2G}{rc^2} \int_0^{r_0} dr' 4\pi r'^2 \rho = \frac{2GM_{total}}{rc^2} \quad (9)$$

Thus, unlike in Newtonian case, we see that in Mannheim's conformal gravity case, the gravitational attraction doesn't depend only on the total mass, but also on the mass distribution. Therefore, if two spherically symmetric objects with the same mass, but different density distributions yield the same strength of gravitational forces, conformal gravity is falsified. On the other hand, if they yield the different strengths of gravitational force, precisely in such a manner the conformal gravity predicts, conformal gravity will be verified. Notice that the difference of the gravitational force would be big; it would be in leading order, not in next to leading order. It could be as big as double or triple. A simple, Cavendish-type experiment would be able to test whether conformal gravity is correct or not. In fact, many Cavendish-type experiments have been performed and none of them has noticed that the gravity depends on the density distribution. Therefore, it seems that conformal gravity is already falsified. Mannheim's conformal gravity may fit the galaxy rotation curve, but it doesn't pass the test of numerous Cavendish-type experiments. Another way of saying is this: if conformal gravity were correct, Cavendish-type experiments should measure α_g not G the Newton constant, as there is no way one can derive the former from the latter. However, one can only measure G in actual experiments, so it seems that Mannheim's conformal gravity is already wrong.

5 Using the Newtonian formula, not the conformal one, to fit the galaxy rotation curve

To fit the galaxy rotation curve, Mannheim applied his formula for the gravitational potential $V(r) = -\beta c^2/r + \gamma c^2 r/2$ to the mass distribution of galaxy. For the first term that contains β , Mannheim calculated as follows, in appendix A of [1]:

$$V_\beta(R, z) = -\beta c^2 \int_0^\infty dR' \int_0^{2\pi} d\phi' \int_{-\infty}^\infty dz' \frac{R' \rho(R', z')}{(R^2 + R'^2 - 2RR' \cos \phi' + (z - z')^2)^{1/2}} \quad (10)$$

However, we can easily see that this equation is a variant of (9), not the variant of (8), as it should be, if Mannheim were faithful to his equations for conformal grav-

ity. (Remember that $4\pi r'^2$ in (9), which comes from $\int d\theta' \int d\phi' R'^2 \sin \theta$ in spherical coordinate, corresponds to $\int d\phi' \int dz' R'$ in the cylindrical coordinate.)

Therefore he lacked consistency. After all, he would not have been able to use the variant of (8) either, since it deviates so much from the Newtonian one, which the conformal one should reduce to.

6 Falsification by the non-existence of unique scale

Given this, one may wonder how Mannheim and Kazanas concluded that conformal gravity reduces to Newtonian gravity in short distances in their paper [3]. It's because they only considered the case when $r' = 0$ in (8); they failed to consider the possibility that the gravitating object could be located somewhere other than the center. They just simply noted that Newtonian $1/r$ potential was reproduced, as if it could be the whole justification.

Actually, they spent a fair amount of their paper on elaborating the case $r' = 0$. (See page 348) They noted that a delta function distribution of the matter located at $r' = 0$ would make (8) zero, but in reality it could be “extended soliton or bag-like objects rather than pointlike ones,” thus making it non-zero. Nevertheless, their argument is useless; equating (8) and (9), their consideration would yield:

$$2\beta = \frac{1}{8\alpha_g} \int_0^{r_0} dr' r'^4 \rho = \frac{2G}{c^2} \int_0^{r_0} dr' 4\pi r'^2 \rho = \frac{2GM_{total}}{c^2} \quad (11)$$

which would suggest that there is a universal length scale for every object, which would be roughly given by $\sqrt{\frac{G\alpha_g}{c^2}}$. However, we know that nucleus of an atom is much larger than an electron, and Earth is much larger than a soccer ball; there can't be such a unique length scale.

7 Falsification by wrong gravitational potential

Mannheim compares the gravitational potential in the Newtonian gravity and the one in the conformal gravity in [1]. He considered the case that all the matter is inside the region ($r < R$), and the mass distribution only depends on r (i.e. spherically symmetric). In case of Newtonian, the potential is given by:

$$\nabla^2 \phi(\vec{r}) = g(\vec{r}) \quad (12)$$

The solution is given by:

$$\phi(r > R) = -\frac{1}{r} \int_0^R dr' r'^2 g(r') \quad (13)$$

$$\phi(r < R) = -\frac{1}{r} \int_0^r dr' r'^2 g(r') - \int_r^R dr' r' g(r') \quad (14)$$

In case of Mannheim's conformal gravity, we have:

$$\nabla^4 \phi(\vec{r}) = h(\vec{r}) \quad (15)$$

The solution is given by:

$$\phi(r > R) = -\frac{1}{6r} \int_0^R dr' r'^4 h(r') - \frac{r}{2} \int_0^R dr' r'^2 h(r') \quad (16)$$

$$\phi(r < R) = -\frac{1}{6r} \int_0^r dr' r'^4 h(r') - \frac{1}{2} \int_r^R dr' r'^3 h(r') - \frac{r}{2} \int_0^r dr' r'^2 h(r') - \frac{r^2}{6} \int_r^R dr' r' h(r') \quad (17)$$

Assuming that conformal gravity reduces to Newtonian gravity in short distances, by comparing the first term of (14) with the first term of (17), we conclude:

$$h(r) = \frac{6g(r)}{r^2} \quad (18)$$

On the other hand, comparing the second term of (14) with the second term of (17), we conclude:

$$h(r) = \frac{2g(r)}{r^2} \quad (19)$$

However, we see that (18) and (19) are different. Therefore, the gravitational potential of conformal gravity doesn't reduce to the Newtonian one. Therefore, conformal gravity is falsified.

Finally, we want to note that Mannheim's conformal gravity we have dealt in this paper should not be confused with Anderson-Barbour-Foster-Murchadha conformal gravity [4].

References

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